Mathematical Analysis - List 5

1. Find a formula for the general term a_n of the sequence, assuming that the pattern of the first few terms continues.

a) $1, \frac{1}{2}, \frac{1}{4}, \frac{1}{8}, \frac{1}{16}, \ldots;$ b) $\frac{1}{2}, \frac{1}{4}, \frac{1}{6}, \frac{1}{8}, \ldots;$ c) $\frac{3}{17}, \frac{4}{26}, \frac{5}{37}, \frac{6}{50}, \ldots;$

d) $1, 3, 5, 7, 9, 11, \ldots;$ e) $15, 0, 15, 0, 15, 0, 15, 0 \ldots;$ f) $1, 4, 27, 256, \ldots.$

2. Use the Limit Laws given in class to find the limit of the sequence.

a) $\lim\limits_{n\to\infty} \left(\sqrt{n^2+4n+1} - \sqrt{n^2+2n}\right);$ b) $\lim\limits_{n\to\infty} \dfrac{n^3+2n^2+1}{n-3n^3};$

c) $\lim\limits_{n\to\infty} \dfrac{(n^2+1)\,n!+1}{(2n+1)(n+1)!};$ d) $\lim\limits_{n\to\infty} \dfrac{\sqrt[3]{8^{n+1}+3}}{2^n+1}.$

3. Use the Squeeze Theorem to prove that

a) $\lim\limits_{n\to\infty} \sqrt[n]{n^3+3n^2+11} = 1;$ b) $\lim\limits_{n\to\infty} \sqrt[n]{\dfrac{7}{n^3}+\dfrac{5}{n^2}+\dfrac{3}{n}} = 1;$

c) $\lim\limits_{n\to\infty} \dfrac{12n+\operatorname{arc\,tg} n}{13n+2} = \dfrac{12}{13};$ d) $\lim\limits_{n\to\infty} \left(\dfrac{1}{\sqrt{n^2+1}} + \dfrac{1}{\sqrt{n^2+2}} + \cdots + \dfrac{1}{\sqrt{n^2+n}}\right) = 1.$

4. Use the Monotonic Sequence Theorem to determine whether the sequence is convergent.

a) $a_n = \dfrac{4n}{n+3};$ b) $a_n = \ln(n+1) - \ln n;$

c) $a_n = \dfrac{1}{n^2-6n+10};$ d) $a_n = \dfrac{4^n}{4^n+3^n};$

e) $a_n = \dfrac{10^n}{n!};$ f) $a_n = \dfrac{5\cdot 7\cdot \ldots \cdot (3+2n)}{4\cdot 7\cdot \ldots \cdot (1+3n)}.$

5. Find the following limits:

a) $\lim\limits_{n\to\infty} \left(\dfrac{5n+2}{5n+1}\right)^{15n};$ b) $\lim\limits_{n\to\infty} \left(\dfrac{3n}{3n+1}\right)^{n};$

c) $\lim\limits_{n\to\infty} \left(1+\dfrac{1}{n}\right)^{3n-2};$ d) $\lim\limits_{n\to\infty} \left(\dfrac{n+4}{n+3}\right)^{5-2n}.$

6. Prove that $\lim\limits_{n\to\infty} -n^3 = -\infty.$

7. Evaluate the following limits:

a) $\lim\limits_{n\to\infty} \left(n^4-3n^3-2n^2-1\right);$ b) $\lim\limits_{n\to\infty} \dfrac{1-(n+1)!}{n!+2};$

c) $\lim\limits_{n\to\infty} \dfrac{n^2+1}{n};$ d) $\lim\limits_{n\to\infty} \dfrac{1+\frac{1}{2}+\ldots+\frac{1}{2^n}}{1+3+\ldots+(2n-1)}.$